Quality Heterogeneity and Misallocation: The Welfare Benefits of Raising your Standards
The Planner's Allocation

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In this addendum, we study the planner's allocation under GTP preferences. We solve for the planner's problem, and then compare the planner allocation to the market allocation. Finally, we compare the social and market allocations under the three specific types of preferences: IA, DA, and homothetic. There results are the basis for the discussion in Section 3.4 in the main text.

1 Planner's Allocation

The planner maximizes the consumer's utility, subject to the resource constraint:

$$\max U = \int_{\Omega} \left(az\xi q(\omega) - \frac{(\xi q(\omega))^{1+\frac{1}{\gamma}}}{1+\frac{1}{\gamma}} \right) d\omega + \frac{\xi^{-\eta} - 1}{\eta}$$
$$\text{s.t. } L = J \int_{\Omega} Lcq(\omega) - Jf_E$$

The first order conditions with respect to $q(z)$ is given by

$$ax\xi - \xi^{1+\frac{1}{\gamma}} q(z)^{\frac{1}{\gamma}} = \delta Lc$$

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where δ is the lagrangian multiplier associated with the resource constraint. The planner's determined cutoff is:

$$z^* = \frac{\delta Lc}{a\xi} \tag{1}$$

Substituting the cutoff into the first order condition yields

$$q(z) = \frac{(az^*)^\gamma}{\xi}\left(\frac{z}{z^*} - 1\right)^\gamma$$

The first order condition with respect to the mass of entrants J is given by:

$$\frac{b^\kappa}{(z^*)^\kappa}\left[\xi\int_{z^*}^{\infty} k\frac{(z^*)^\kappa}{z^{\kappa+1}} zq(z)dz - \frac{\gamma}{1+\gamma}\int_{z^*}^{\infty} k\frac{(z^*)^\kappa}{z^{\kappa+1}}(\xi q(z))^{1+\frac{1}{\gamma}}dz - \delta Lc\int_{z^*}^{\infty} k\frac{(z^*)^\kappa}{z^{\kappa+1}}q(z)dz\right] = \delta f_E$$

$$\frac{b^\kappa (az^*)^{1+\gamma}}{(z^*)^\kappa}\left[G_3(1) - \frac{\gamma}{1+\gamma}G_1(1) - G_4(1)\right] = \frac{f_E\xi(az^*)}{Lc}$$

where $G_4(1)$ is given by:

$$G_4(1) = \int_{z^*}^{\infty} \kappa\left(\frac{z}{z^*} - 1\right)^\gamma \frac{(z^*)^\kappa}{z^{\kappa+1}}dz = \frac{\kappa F_2(1)}{\kappa - \gamma} \tag{2}$$

Using G_1 and G_3 from the main text, and (2), we obtain that:

$$G_3(1) - \frac{\gamma}{1+\gamma}G_1(1) - G_4(1) = \frac{G_1(1)}{1+\gamma}$$

Thus, the first order condition with respect to the mass of entrants becomes:

$$\frac{b^\kappa (az^*)^\gamma}{(z^*)^\kappa}\frac{G_1(1)}{1+\gamma} = \frac{f_E\xi}{Lc} \tag{3}$$

Substituting (3) into the resource constraint yields:

$$L = J\left[\frac{Lcb^k}{(z^*)^\kappa}\int_{z^*}^{\infty} k\frac{(z^*)^\kappa}{z^{\kappa+1}}q(z)dz + f_E\right]$$

$$L = J\left[\frac{Lcb^k(az^*)^\gamma}{\xi(z^*)^\kappa}G_4(g) + f_E\right]$$

$$L = Jf_E\left[\frac{(1+\gamma)G_4(g)}{G_1(1)} + 1\right]$$

$$J = \frac{L}{f_E}\frac{G_1(1)}{G_2(1)} = \frac{L}{f_E(\kappa - \gamma)} \tag{4}$$

where the result is obtained as:

$$\frac{F_1(1)}{F_2(1)} = \frac{\kappa}{\kappa - \gamma} \tag{5}$$

and

$$\frac{G_1(1)}{G_2(1)} = \frac{\frac{F_1(1)}{F_2(1)(\kappa-\gamma-1)} - \frac{1}{\kappa-\gamma}}{\frac{F_1(1)}{F_2(1)(\kappa-\gamma-1)} + \frac{\gamma}{\kappa-\gamma}} = \frac{1}{\kappa - \gamma}$$

Using (**??**) and (**??**) into the definition of ξ (**??**) we obtain:

$$\begin{aligned}
\xi^{-\eta} &= \frac{Jb^{\kappa}}{(z^*)^{\kappa}} \left[\xi \int_{z^*}^{\infty} k \frac{(z^*)^{\kappa}}{z^{\kappa+1}} zq(z)dz - \int_{z^*}^{\infty} k \frac{(z^*)^{\kappa}}{z^{\kappa+1}} (\xi q(z))^{1+\frac{1}{\gamma}} dz \right] = \\
&= \frac{Jb^{\kappa}(az^*)^{1+\gamma}}{(z^*)^{\kappa}} \left[G_3(1) - G_1(1) \right] = \\
&= \frac{Jb^{\kappa}(az^*)^{1+\gamma}}{(z^*)^{\kappa}} G_4(1)
\end{aligned}$$

Substituting the result into (3) yields the planner's determined cutoff:

$$z^* = \left[\frac{Jb^k c^{\frac{\eta}{1+\eta}} a^{\gamma + \frac{1}{1+\eta}}}{(1+\gamma)^{\frac{\eta}{1+\eta}}} G_4(1)^{\frac{1}{1+\eta}} G_2(1)^{\frac{\eta}{1+\eta}} \right]^{\frac{1}{\kappa - \gamma - \frac{1}{1+\eta}}}$$

2 Planner's and Market's Allocations

Let us now compare the market allocation with the planner's allocation. To facilitate the comparison, we use subscript m to denote variables in the market allocation, and subscript p in the planner's allocation.

Entry. The planner chooses a mass of entrants identical to the mass of entrants in the market allocation. In fact, letting $J_m(g)$ denote the mass of entrants in the market allocation as a function of the quality standard, and let $J_p = J_m(1)$ denote the planner's entry decision. The ratio of the two measures of entry equals 1 in the absence of a standard, and it is increasing in the restrictiveness of the standard:

$$\begin{aligned}
\frac{J_m(g)}{J_p} &= \frac{G_1(g)G_2(1)}{G_2(g)G_1(g)} \qquad g \geq 1 \\
\frac{J_m(1)}{J_p} &= 1 \qquad g = 1
\end{aligned} \tag{6}$$

Selection. Let us re-write the market quality cutoff as a function of the standard:

$$z_m^*(g) = \left[\frac{J_m(g) c^{\frac{\eta}{1+\eta}} \gamma^\gamma b^\kappa a^{\gamma + \frac{1}{1+\eta}}}{(1+\gamma)^{1+\gamma}} g^{-\kappa} G_2(g) \right]^{\frac{1}{\kappa - \gamma - \frac{1}{1+\eta}}}$$

where we used the solution for entry from the main text. Dividing the planner's cutoff by the market cutoff yields:

$$\frac{z_p^*}{z_m^*(g)} = \left[\left(1 + \frac{1}{\gamma}\right)^\gamma \frac{J_p}{J_m(g)} g^\kappa \frac{((1+\gamma)G_4(1))^{\frac{1}{1+\eta}} G_2(1)^{\frac{\eta}{1+\eta}}}{G_2(g)} \right]^{\frac{1}{\kappa - \gamma - \frac{1}{1+\eta}}} \tag{7}$$

Evaluating the ratio in the market allocation yields:

$$\frac{z_p^*}{z_m^*(1)} = \left(1 + \frac{1}{\gamma}\right)^{\frac{\gamma(1+\eta)}{(1+\eta)(\kappa-\gamma)-1}} \left(1 - \frac{1}{\kappa - \gamma}\right)^{\frac{1}{(1+\eta)(\kappa-\gamma)-1}} \tag{8}$$

where we used (5).

Market Aggregator. Let us now focus on the aggregator ξ. Using the zero profit condition from the market allocation, we obtain:

$$\begin{aligned} \xi_m(g)(z_m^*(g))^{\kappa-\gamma} &= \frac{Lc}{f_E(1+\gamma)} \left(\frac{a\gamma}{1+\gamma}\right)^\gamma \frac{b^\kappa}{g^\kappa} G_1(g) = \\ &= \frac{c}{f_E(1+\gamma)} \left(\frac{a\gamma}{1+\gamma}\right)^\gamma \frac{b^\kappa}{g^\kappa} G_2(g) J_m(g) \end{aligned}$$

Using the first order condition with respect to the mass of entrants (3) from the planner's problem yields:

$$\xi_p (z_p^*)^{\kappa-\gamma} = \frac{c b^\kappa a^\gamma}{f_E(1+\gamma)} J_p G_2(1)$$

Taking the ratio of the two expressions yields:

$$\frac{\xi_p}{\xi_m(g)} = \left(1 + \frac{1}{\gamma}\right)^\gamma \left(\frac{z_m^*(g)}{z_p^*}\right)^{\kappa-\gamma} \frac{J_p}{J_m(g)} \frac{G_2(1)}{G_2(g)} g^\kappa \tag{9}$$

Evaluating it at $g = 1$ yields:

$$\frac{\xi_p}{\xi_m(1)} = \left(1 + \frac{1}{\gamma}\right)^\gamma \left(\frac{z_m^*(1)}{z_p^*}\right)^{\kappa-\gamma} \tag{10}$$

Markups. Let us consider the constant social markup as in Bertoletti and Etro (2018). To obtain an optimal allocation, in fact, it suffices for all varieties to have the same constant markup m, so that the marginal rate of substitutions equal the marginal rate of transformation. To obtain the markup m let us start from the budget constraint. As $p(z) = mc$, it follows that:

$$J\int_{z_p^*}^{\infty} p(z)q(z) = mJ\int_{z_p^*}^{\infty} cq(z)$$

Substituting the result into the budget constraint yields:

$$m = \frac{\kappa - \gamma}{\kappa - \gamma - 1} \tag{11}$$

3 Comparison Across Preferences

We compare the market allocation under a standard with the planner's allocation, under IA preferences (figure 1), DA preferences (figure 2), and homothetic preferences with too little selection in the market (figure 3) and too much selection (figure 4). Across preferences, we focus on the ratios $\frac{\xi_p}{\xi_m(g)}$ and $\frac{z_p^*}{z_m^*(g)}$ as a function of g. Furthermore, we compare markups and quantitites under the market allocation, the market allocation under the optimal standard, and the planner's allocation. To derive markups and quantities in the market allocation, we normalize the planner's cutoff, entry, and aggregator to one, and derive the corresponding market variables using (10), (8), and (6).

Figure 1: IA Preferences: Market VS Planner

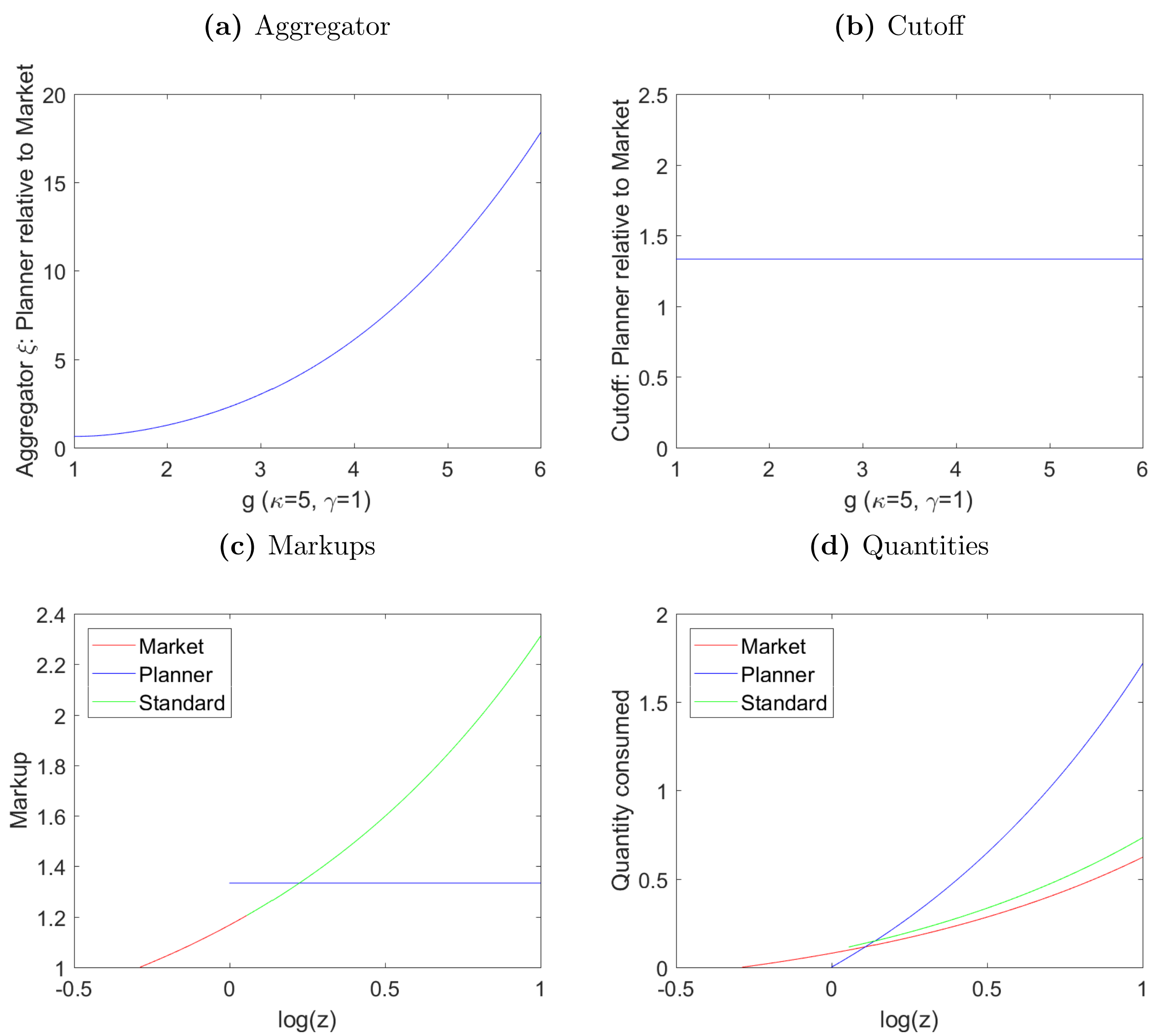
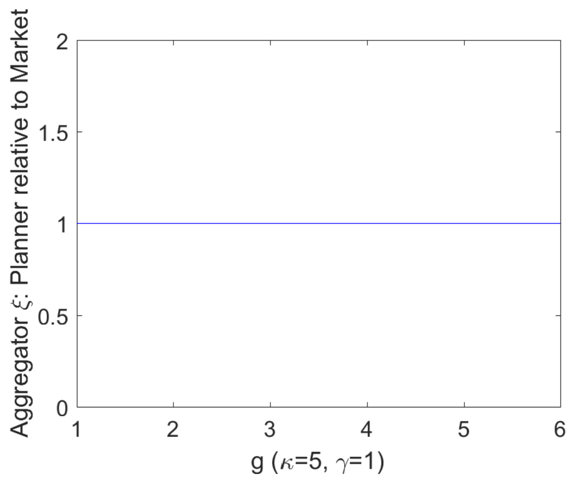
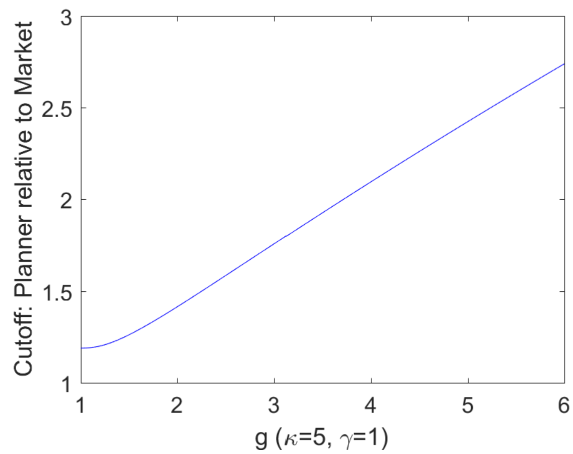


Figure 2: DA Preferences: Market VS Planner

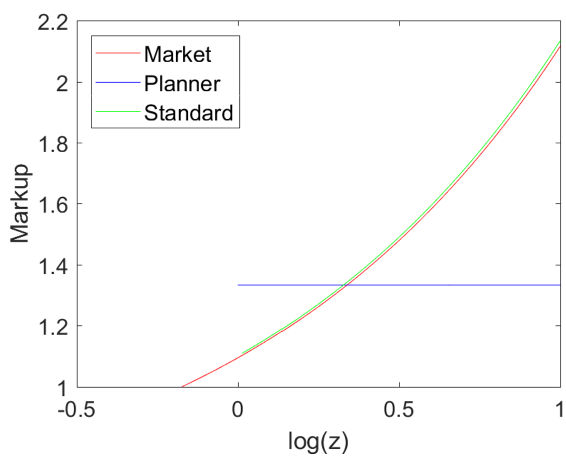
(a) Aggregator



(b) Cutoff



(c) Markup



(d) Quantities

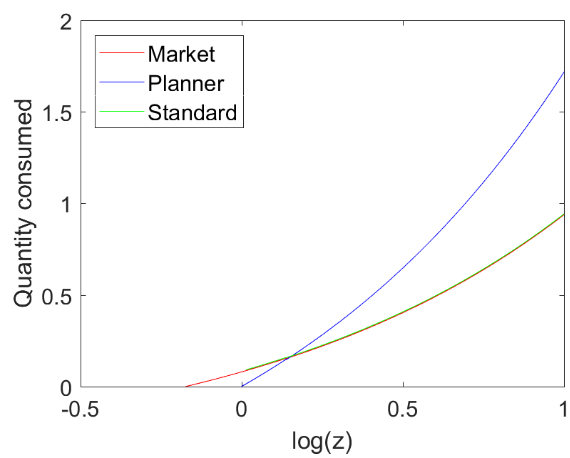
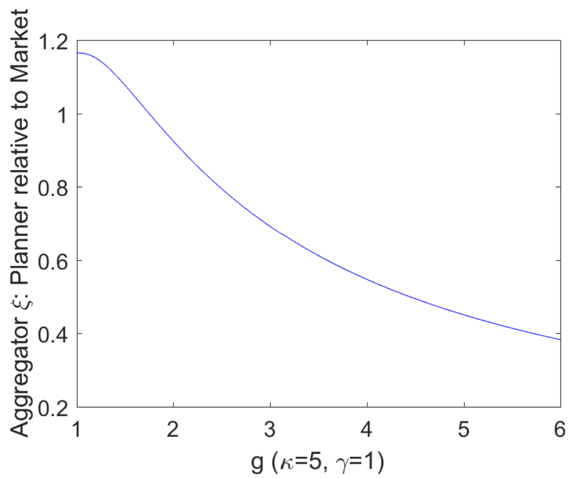
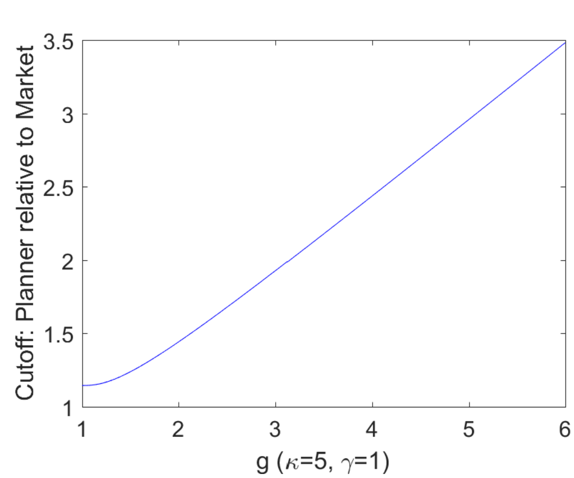


Figure 3: Homothetic Preferences: Market VS Planner (Too little selection)

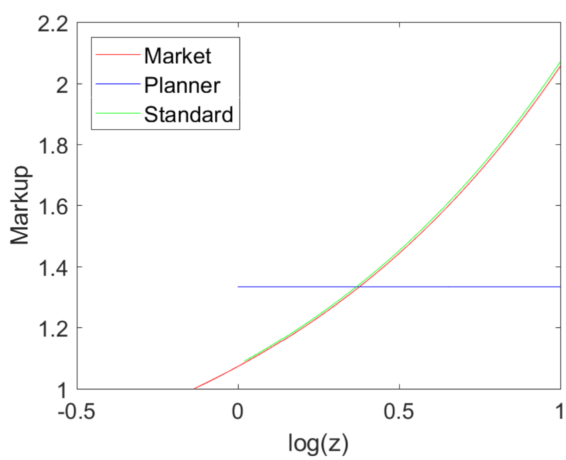
(a) Aggregator



(b) Cutoff



(c) Markup



(d) Quantities

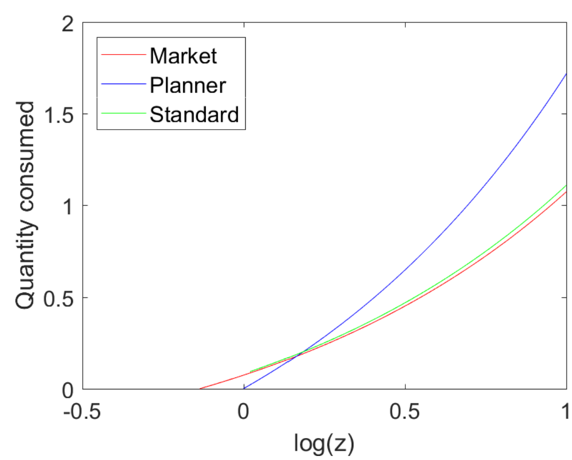
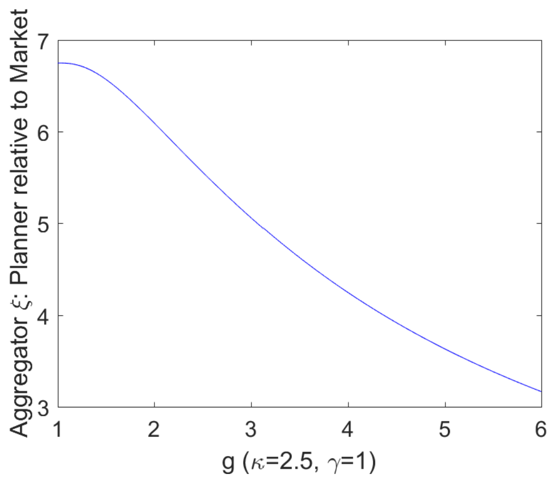
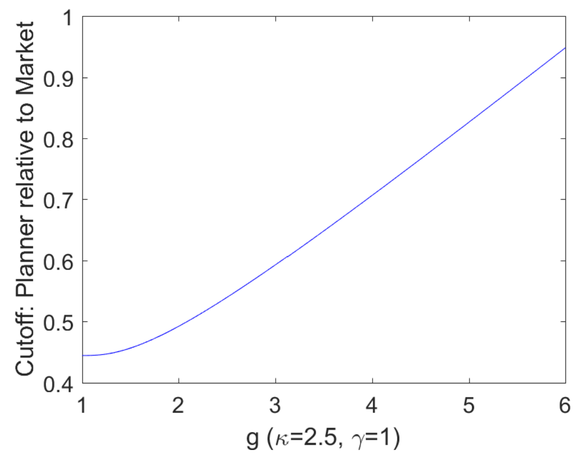


Figure 4: Homothetic Preferences: Market VS Planner (Too much selection)

(a) Aggregator



(b) Cutoff



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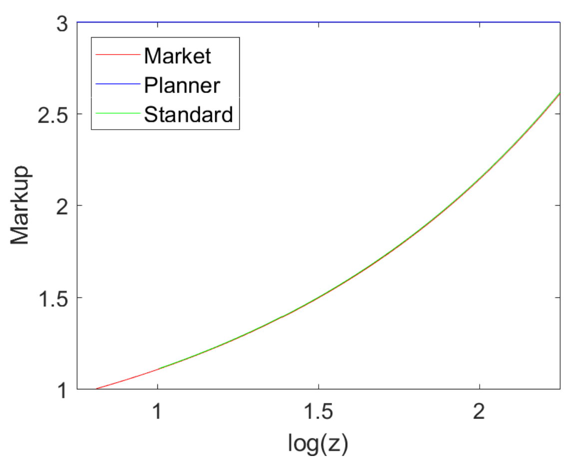
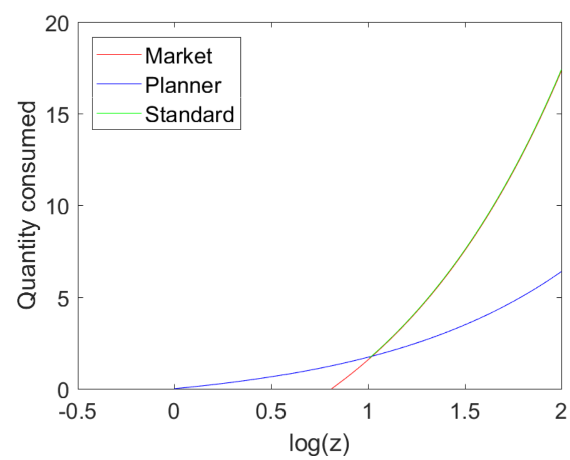


(d) Quantities



References

P. Bertoletti and F. Etro. Monopolistic Competition with GAS Preferences. *DEM Working Paper Series*, (165), 2018.