

# Quality Heterogeneity and Misallocation: The Welfare Benefits of Raising your Standards Other VES Preferences

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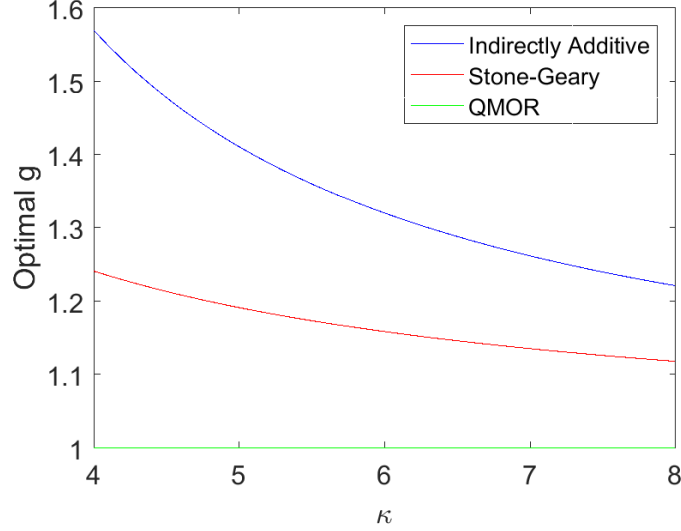
In this addendum, we study the welfare effects of a quality standard under three preference specifications not included under the GTP class with variable elasticity of substitution (VES). In particular, we consider a case of the indirectly additive preferences analyzed in [Bertoletti et al. \(2017\)](#), which we label "addilog", the Stone-Geary preferences [Simonovska \(2015\)](#), which are directly additive, and the Quadratic Mean of Order R (QMOR) expenditure function proposed by [Feenstra \(2014\)](#). The ranking of preferences in terms of level of optimal standard is respected by these three preferences, as shown in figure (1). Addilog yields the largest optimal standard, Stone-Geary yields the intermediate value, and QMOR yields the lowest value (equal to one). In the following paragraphs we show how to derive the welfare effects of a standard under these three preferences.

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**Figure 1:** Preferences and Optimal Standard



# 1 Addilog

## 1.1 Consumers' Problem

Preferences of consumers in country  $j$  are given by the following indirectly additive utility function

$$U_j = \frac{\left[ a \int_{\Omega_j} z(\omega) x^c(\omega) d\omega - 1 \right]^{1+\gamma}}{(1+\gamma) \left( \int_{\Omega_j} (x^c(\omega))^{(1+\gamma)/\gamma} d\omega \right)^\gamma} \quad (1)$$

Maximizing the utility subject to a budget constraint yields the first order conditions with respect to  $x^c(\omega)$ :

$$\left[ \frac{a \int_{\Omega_j} z(\omega) x^c(\omega) d\omega - 1}{\int_{\Omega_j} (x^c(\omega))^{(1+\gamma)/\gamma} d\omega} \right]^\gamma a z(\omega) - \left[ \frac{a \int_{\Omega_j} z(\omega) x^c(\omega) d\omega - 1}{\int_{\Omega_j} (x^c(\omega))^{(1+\gamma)/\gamma} d\omega} \right]^{1+\gamma} (x^c(\omega))^{\frac{1}{\gamma}} = \lambda_j p_j(\omega)$$

where  $\lambda_j$  is the marginal utility of income and  $p_j(\omega)$  is the price of the  $\omega$  variety. Multiplying both sides of the first order condition by  $x^c(\omega)$  and integrating over the set of varieties  $\Omega_j$  yields an expression for the marginal utility of income:

$$\lambda_j = \frac{1}{y_j} \left[ \frac{a \int_{\Omega_j} z(\omega) x^c(\omega) d\omega - 1}{\int_{\Omega_j} (x^c(\omega))^{(1+\gamma)/\gamma} d\omega} \right]^\gamma \quad (2)$$

Substituting (2) into the first order condition yields the following inverse demand function

$$p_j(\omega) = ay_j z(\omega) - (\lambda_j x^c(\omega))^{\frac{1}{\gamma}} (y_j)^{(1+\gamma)/\gamma}$$

Aggregate demand for a variety  $\omega$  is  $x(\omega) = L_j x^c(\omega)$ . Hence, the aggregate inverse demand equals

$$p_j(\omega) = ay_j z(\omega) - \left(\frac{\lambda_j}{L_j}\right)^{\frac{1}{\gamma}} (y_j)^{(1+\gamma)/\gamma} x(\omega)^{\frac{1}{\gamma}} \quad (3)$$

Finally, using (2) into (1) yields an easier expression for the utility of consumers

$$U_j = \frac{\lambda_j y_j}{1 + \gamma} \left[ a \int_{\Omega_j} z(\omega) x^c(\omega) d\omega - 1 \right] \quad (4)$$

## 1.2 Firm's Problem

The profit of a firm with quality  $z$  in a destination  $j$ , using (3), are given by:

$$\pi_{ij}(z) = p_{ij}(z)x_{ij}(z) - w_i c_{ij} x_{ij}(z) = ay_j z x_{ij}(z) - \left(\frac{\lambda_j}{L_j}\right)^{\frac{1}{\gamma}} (y_j x_{ij}(z))^{(1+\gamma)/\gamma} - w_i c_{ij} x_{ij}(z) \quad (5)$$

where  $w_i$  is the labor wage in  $i$  and  $c_{ij}$  is the marginal cost of production and delivery. Profit maximization with respect to  $x_{ij}(z)$  yields the following first order condition:

$$ay_j z - \left(1 + \frac{1}{\gamma}\right) \left(\frac{\lambda_j}{L_j}\right)^{\frac{1}{\gamma}} (y_j)^{(1+\gamma)/\gamma} x(\omega)^{\frac{1}{\gamma}} - w_i c_{ij} = 0$$

Evaluating the first order condition at zero output yields the quality cutoff  $z_{ij}^*$ :

$$z_{ij}^* = \frac{w_i c_{ij}}{ay_j} \quad (6)$$

Using (6) we obtain the optimal quantity supplied by a firm with quality  $z$ :

$$x_{ij}(z) = \left(\frac{a\gamma}{1 + \gamma}\right)^{\gamma} \frac{L_j}{\lambda_j y_j} (z - z_{ij}^*)^{\gamma} \quad (7)$$

Substituting (7) into (3) yields the pricing rule

$$p_{ij} = \frac{a\gamma y_j}{1 + \gamma} \left(\frac{z + \gamma z_{ij}^*}{\gamma}\right) = \frac{w_i c_{ij}}{1 + \gamma} \left(\frac{z + \gamma z_{ij}^*}{z_{ij}^*}\right) \quad (8)$$

Firm's revenues are then given by

$$\begin{aligned} r_{ij}(z) &= \frac{L_j}{\lambda_j} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} \left( \frac{z + \gamma z_{ij}^*}{\gamma} \right) (z - z_{ij}^*)^\gamma = \\ &= \frac{L_j}{\lambda_j} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} \left( \frac{1}{\gamma} + \frac{z_{ij}^*}{z} \right) \left( 1 - \frac{z_{ij}^*}{z} \right)^\gamma z^{1+\gamma} \end{aligned} \quad (9)$$

and profits equal

$$\pi_{ij}(z) = \frac{L_j}{\lambda_j} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} (z - z_{ij}^*)^{1+\gamma} = \quad (10)$$

$$= \frac{L_j}{\lambda_j} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} \left( 1 - \frac{z_{ij}^*}{z} \right)^{1+\gamma} z^{1+\gamma} \quad (11)$$

### 1.3 Equilibrium

Let us consider a closed economy. Thus, we drop subscripts  $i$  and  $j$  and normalize per capita income  $y_j = w_j = 1$ . The government establishes a minimum quality standard  $\bar{z} \geq z^*$  such that any firm with quality  $z < \bar{z}$  is forced out of the market. It is convenient to express the restrictiveness of the quality standard as  $g = \frac{\bar{z}}{z^*} \geq 1$ .

The mass of firms selling to the economy is

$$N = J \left( \frac{b}{\bar{z}} \right)^\kappa = J \left( \frac{b}{gz^*} \right)^\kappa \quad (12)$$

where  $J$  is the mass of entrants,  $b$  is the shift parameter of the Pareto distribution of quality and  $\kappa$  is the shape parameter of the distribution of quality.

Average profits equal:

$$\begin{aligned} \bar{\pi} &= \frac{L}{\lambda} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} \int_{\bar{z}}^{\infty} \left( 1 - \frac{z^*}{z} \right)^{1+\gamma} z^{\gamma-\kappa} \kappa \bar{z}^\kappa dz = \\ &= \frac{L}{\lambda} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} \left[ \frac{\kappa \bar{z}^{1+\gamma} F_1(g)}{\kappa - \gamma - 1} - \frac{\kappa \bar{z}^\gamma z^* F_2(g)}{\kappa - \gamma} \right] = \\ &= \frac{L}{\lambda} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} \bar{z}^{1+\gamma} \kappa \left[ \frac{F_1(g)}{\kappa - \gamma - 1} - g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right] \end{aligned}$$

We restrict the parameter space such that  $\kappa - \gamma - 1 > 0$ .  $F_1(g)$  and  $F_2(g)$  are two hypergeometric functions given by:

$$F_1(g) = {}_2F_1 [\kappa - \gamma - 1, -\gamma; \kappa - \gamma, g^{-1}] \quad (13)$$

$$F_2(g) = {}_2F_1[\kappa - \gamma, -\gamma; \kappa - \gamma + 1, g^{-1}] \quad (14)$$

Setting the expected profits equal to the fixed cost of entry yields the equilibrium marginal utility of income

$$\begin{aligned} f_E &= \left(\frac{b}{\bar{z}}\right)^\kappa \bar{\pi} \\ f_E &= \frac{L}{\lambda} \left(\frac{a\gamma}{1+\gamma}\right)^{1+\gamma} \frac{\kappa b^\kappa}{\bar{z}^{\kappa-\gamma-1}} \left[ \frac{F_1(g)}{\kappa - \gamma - 1} - g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right] \\ \lambda &= \frac{L}{f_E} \left(\frac{a\gamma}{1+\gamma}\right)^{1+\gamma} \frac{\kappa b^\kappa}{\bar{z}^{\kappa-\gamma-1}} \left[ \frac{F_1(g)}{\kappa - \gamma - 1} - g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right] \\ \lambda &= \frac{L}{f_E} \left(\frac{a\gamma}{1+\gamma}\right)^{1+\gamma} \frac{\kappa b^\kappa}{(z^*)^{\kappa-\gamma-1}} g^{-(\kappa-\gamma-1)} \left[ \frac{F_1(g)}{\kappa - \gamma - 1} - g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right] \end{aligned} \quad (15)$$

Since the quality cutoff is a constant (6):  $z^* = c/a$ , the quality standard only affects the marginal utility of income through  $g$ .

Similarly, average revenues are given by

$$\bar{r} = \frac{L}{\lambda} \left(\frac{a\gamma}{1+\gamma}\right)^{1+\gamma} \bar{z}^{1+\gamma} \frac{\kappa}{\gamma} \left[ \frac{F_1(g)}{\kappa - \gamma - 1} + \gamma g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right]$$

Setting aggregate revenues equal to aggregate income  $L$  yields:

$$\begin{aligned} \frac{L}{\lambda} \left(\frac{a\gamma}{1+\gamma}\right)^{1+\gamma} \frac{J\kappa b^\kappa}{\bar{z}^{\kappa-\gamma-1}} \left[ \frac{F_1(g)}{\kappa - \gamma - 1} + \gamma g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right] &= L \\ \frac{a}{\lambda} \left(\frac{a\gamma}{1+\gamma}\right)^\gamma \frac{J\kappa b^\kappa}{\bar{z}^{\kappa-\gamma-1}} &= (1+\gamma) \left[ \frac{F_1(g)}{\kappa - \gamma - 1} + \gamma g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right]^{-1} \end{aligned} \quad (16)$$

which is a result we are going to use in the next derivation. Finally, we need to derive the following integral:

$$a \int_{\bar{z}}^{\infty} z x^c(z) dz - 1 = \frac{a}{\lambda} \left(\frac{a\gamma}{1+\gamma}\right)^\gamma \frac{J\kappa b^\kappa}{\bar{z}^{\kappa-\gamma-1}} \frac{F_1(g)}{\kappa - \gamma - 1} - 1$$

Using (16), we obtain

$$\begin{aligned} a \int_{\bar{z}}^{\infty} z x^c(z) dz - 1 &= \frac{(1+\gamma)F_1(g)}{\kappa - \gamma - 1} \left[ \frac{F_1(g)}{\kappa - \gamma - 1} + \gamma g^{-1} \frac{F_2(g)}{\kappa - \gamma} \right]^{-1} = \\ &= \gamma \frac{\frac{F_1(g)}{\kappa - \gamma - 1} - g^{-1} \frac{F_2(g)}{\kappa - \gamma}}{\frac{F_1(g)}{\kappa - \gamma - 1} + \gamma g^{-1} \frac{F_2(g)}{\kappa - \gamma}} \end{aligned} \quad (17)$$

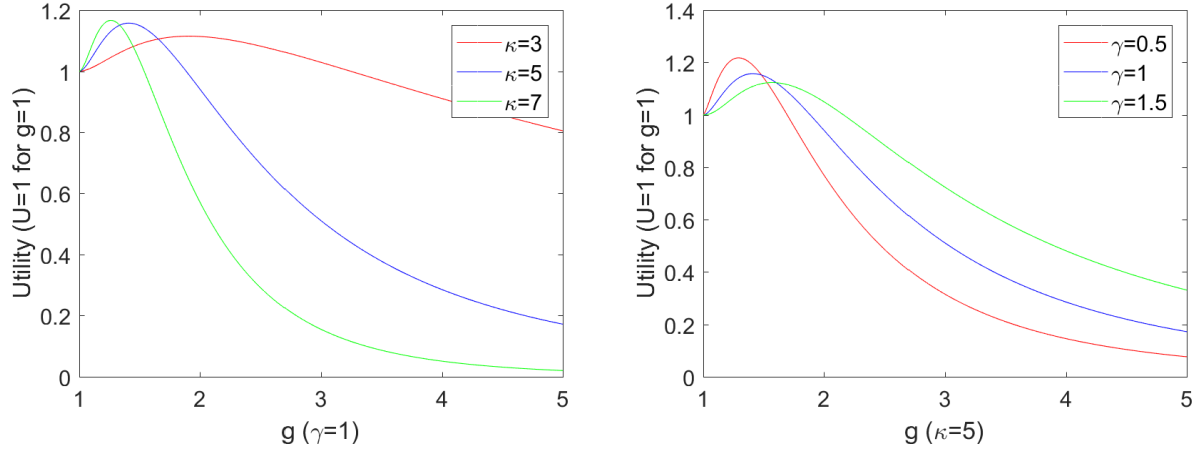
Thus, using (15) (17) into (4), the utility of the representative consumer is given by

$$\begin{aligned}
 U &= \frac{\lambda}{1+\gamma} \left[ a \int_{\bar{z}}^{\infty} z x^c(z) dz - 1 \right] = \\
 &= \frac{\gamma}{1+\gamma} \frac{L}{f_E} \left( \frac{a\gamma}{1+\gamma} \right)^{1+\gamma} \frac{\kappa b^\kappa}{(z^*)^{\kappa-\gamma-1}} g^{-(\kappa-\gamma-1)} \frac{\left[ \frac{F_1(g)}{\kappa-\gamma-1} - g^{-1} \frac{F_2(g)}{\kappa-\gamma} \right]^2}{\frac{F_1(g)}{\kappa-\gamma-1} + \gamma g^{-1} \frac{F_2(g)}{\kappa-\gamma}}
 \end{aligned} \tag{18}$$

## 1.4 Welfare and Minimum Quality Standard

Figure 2 illustrates the relationship between utility (18) and  $g$  for different sets of parameters. Higher values of  $\gamma$  increases the optimal quality standard, while higher values of  $\kappa$  reduce it.

**Figure 2:** Welfare and Minimum Quality Standard



## 2 Stone-Geary Preferences

Let us consider a closed economy where  $L$  consumers have the following Stone-Geary preferences (Simonovska, 2015):

$$U = \int_{\Omega} u(\omega) d\omega = \int_{\Omega} \ln \left[ \frac{z(\omega) x^c(\omega)}{\bar{q}} + 1 \right] d\omega \tag{19}$$

where all variables have the same interpretation of the previous case, and  $\bar{q}$  is a positive constant. Moreover, let us spare some notation by setting wages and per capita income equal to one. Solving the consumers problem, the inverse aggregate demand function equals:

$$p(\omega) = \frac{L}{\lambda} \frac{z(\omega)}{z(\omega) x(\omega) + L \bar{q}} \tag{20}$$

A firm with quality  $z$  maximizes the following profits, by choosing quantity  $x(z)$  taking the marginal utility of income as given:

$$\pi(z) = \frac{L}{\lambda} \frac{zx(z)}{zx(z) + L\bar{q}} - cx(z)$$

Setting the optimal quantity produced by a firm to zero yields the market determined quality cutoff:

$$z^* = \bar{q}c\lambda \quad (21)$$

Relative to the IA case, the quality cutoff depends on a general equilibrium variable — the marginal utility of income. As a result, markups do not just depend on real income, but also on the number of varieties available for consumption.

The performance variables of firms are given by:

$$\begin{aligned} x(z) &= \frac{L\bar{q}}{z} \left[ \left( \frac{z}{z^*} \right)^{\frac{1}{2}} - 1 \right] \\ p(z) &= c \left( \frac{z}{z^*} \right)^{\frac{1}{2}} \\ r(z) &= \frac{L\bar{q}c}{z^*} \left[ 1 - \left( \frac{z^*}{z} \right)^{\frac{1}{2}} \right] \\ \pi(z) &= \frac{L\bar{q}c}{z^*} \left[ 1 - 2 \left( \frac{z^*}{z} \right)^{\frac{1}{2}} + \frac{z^*}{z} \right] \end{aligned}$$

The mass of firms active in the economy is given by:

$$N = J \left( \frac{b}{\bar{z}} \right)^{\kappa} = J \left( \frac{b}{z^*} \right)^{\kappa} g^{-\kappa}$$

Average revenues, profits and utility, conditional on varieties being sold are given by:

$$\begin{aligned} \bar{r} &= \frac{L\bar{q}c}{z^*} \left[ 1 - \frac{\kappa}{\kappa + 0.5} g^{-0.5} \right] \\ \bar{\pi} &= \frac{L\bar{q}c}{z^*} \left[ 1 - \frac{2\kappa}{\kappa + 0.5} g^{-0.5} + \frac{\kappa}{\kappa + 1} g^{-1} \right] \\ \bar{u} &= \frac{1}{2} \left[ \frac{1}{\kappa} + \ln g \right] \end{aligned}$$

Similarly to the IA case, the government restriction improves the average quality in the

economy — and thus the average utility of consumers. In fact, consumer's utility equals:

$$U = \frac{N}{2} \left[ \frac{1}{\kappa} + \ln g \right] \quad (22)$$

In this case, love for variety and business stealing bias are both represented within  $N$ . Using the market clearing condition, the mass of varieties available for consumption becomes:

$$N = \frac{L}{\bar{r}} = \frac{z^*}{\bar{q}c} \left[ 1 - \frac{\kappa}{\kappa + 0.5} g^{-0.5} \right]^{-1} \quad (23)$$

Using the expected zero profit condition, yields an expression for the market cutoff  $z^*$ :

$$\begin{aligned} z^* &= \left( \frac{L\bar{q}cb^\kappa}{f_E} \right)^{\frac{1}{\kappa+1}} g^{-\frac{\kappa}{\kappa+1}} \left[ 1 - \frac{2\kappa}{\kappa + 0.5} g^{-0.5} + \frac{\kappa}{\kappa + 1} g^{-1} \right]^{\frac{1}{\kappa+1}} \\ &= \left( \frac{L\bar{q}cb^\kappa}{f_E} \right)^{\frac{1}{\kappa+1}} g^{-1} \left[ g - \frac{2\kappa}{\kappa + 0.5} g^{0.5} + \frac{\kappa}{\kappa + 1} \right]^{\frac{1}{\kappa+1}} \end{aligned}$$

Thus,  $N$  becomes:

$$N = \frac{L}{\bar{r}} = \frac{(L\bar{q}cb^\kappa)^{\frac{1}{\kappa+1}} \left[ g - \frac{2\kappa}{\kappa+0.5} g^{0.5} + \frac{\kappa}{\kappa+1} \right]^{\frac{1}{\kappa+1}}}{\bar{q}c f_E^{\frac{1}{\kappa+1}} g - \frac{\kappa}{\kappa+0.5} g^{0.5}} \quad (24)$$

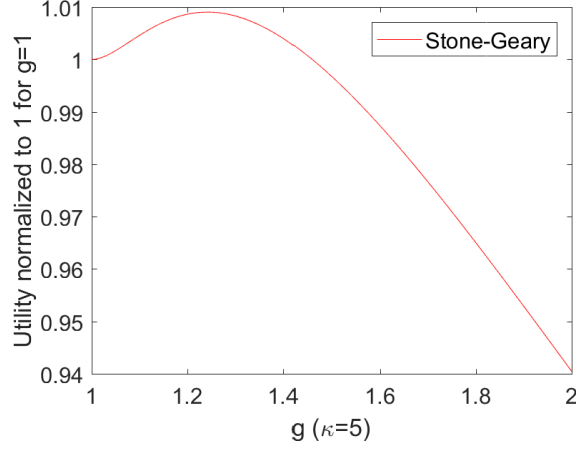
Hence, our utility becomes:

$$U = \frac{(L\bar{q}cb^\kappa)^{\frac{1}{\kappa+1}} \left[ g - \frac{2\kappa}{\kappa+0.5} g^{0.5} + \frac{\kappa}{\kappa+1} \right]^{\frac{1}{\kappa+1}}}{2\bar{q}c f_E^{\frac{1}{\kappa+1}} g - \frac{\kappa}{\kappa+0.5} g^{0.5}} \left[ \frac{1}{\kappa} + \ln g \right] \quad (25)$$

As in the baseline case, there is a level of  $g > 1$  that maximizes welfare: a small quality requirement improves the utility of consumers.



**Figure 3:** Minimum Requirement and Welfare



In the Stone-Geary case we can decompose utility to get a closer look as to why standards raise welfare.

The average markup in the economy is:

$$\bar{\mu} = \frac{\int_{z^*}^{\infty} \mu_{ij} r_{ij}(z) \kappa \frac{b^\kappa}{z^{\kappa+1}} dz}{\int_{z^*}^{\infty} r_{ij}(z) \kappa \frac{b^\kappa}{z^{\kappa+1}} dz} = \frac{g - \frac{2\kappa}{\kappa+0.5} g^{0.5} + \frac{\kappa}{\kappa+1}}{g - \frac{\kappa}{\kappa+0.5} g^{0.5}} \quad (26)$$

which is increasing in  $g$ . Thus, the number of varieties available for consumption can be written as:

$$N = \frac{L}{\bar{r}} = \frac{(L\bar{q}cb^\kappa)^{\frac{1}{\kappa+1}}}{\bar{q}cf_E^{\frac{1}{\kappa+1}}} \bar{\mu} \left[ g - \frac{2\kappa}{\kappa+0.5} g^{0.5} + \frac{\kappa}{\kappa+1} \right]^{-\frac{\kappa}{\kappa+1}} \quad (27)$$

Hence, following the decomposition we used in the addilog case, the utility becomes:

$$U = \frac{L}{\bar{r}} = \frac{(L\bar{q}cb^\kappa)^{\frac{1}{\kappa+1}}}{2\bar{q}cf_E^{\frac{1}{\kappa+1}}} \underbrace{\bar{\mu}}_{\text{Business Stealing Bias}} \underbrace{\left[ g - \frac{2\kappa}{\kappa+0.5} g^{0.5} + \frac{\kappa}{\kappa+1} \right]^{-\frac{\kappa}{\kappa+1}}}_{\text{Love for Variety}} \underbrace{\bar{u}}_{\text{Average Quality}} \quad (28)$$

### 3 Quadratic Mean of Order R (QMOR) Expenditure

From the general case of QMOR, we solve the special case where  $r = 2$  in the expenditure function described in [Feenstra \(2014\)](#).<sup>1</sup> In order to write the demand function we follow [Arkolakis et al. \(2017\)](#), but write it as a quality-adjusted demand, where  $z$  is a measure of

<sup>1</sup>Translog is another special case, where  $r \rightarrow 0$ .

quality:

$$x(z) = \gamma Q \left( \frac{p(z)}{zP} - 1 \right). \quad (29)$$

$P$  is the choke price, as defined in [Feenstra \(2014\)](#), and  $Q$  is a constant taken as given by the firms and defined such that the budget constraint holds.  $Q$  is a function of total expenditure which we solve below. Notice that that  $z$  is a variety specific demand shifter which we interpret as quality.  $\gamma$  is a parameter that governs the love of variety in the expenditure function. We solve for the indirect demand given (29), multiply by quantity to get profits, then solve for prices and quantity:

$$\begin{aligned} \pi(z) &= \frac{x(z)^2 z P}{Q \gamma} + z P x(z) - x(z) c \\ p(z) &= \frac{1}{2} [P z + c] \\ x(z) &= L Q \gamma \frac{1}{2} \left( \frac{c}{P z} - 1 \right) \end{aligned}$$

Now we can set  $x(z) = 0$  above to get:

$$z^* = \frac{c}{P} \quad (30)$$

Then profit as a function of  $z$  and  $z^*$ :

$$\pi(z) = \frac{1}{4} L Q c \gamma \left( -\frac{z^*}{z} - \frac{z}{z^*} - 2 \right) \quad (31)$$

$$r(z) = \frac{1}{4} L Q c \gamma \left( \frac{z^*}{z} - \frac{z}{z^*} \right) \quad (32)$$

The following aggregate statistics are then derived using the Pareto assumption for firm

quality and integrating over  $z \in (z^*, \infty)$ :

$$\begin{aligned} P &= \left[ \frac{N}{N - (\tilde{N} + \gamma/\beta)} \right] \int_{z^*}^{\infty} \frac{1}{N} p(z) dz \\ &= \left[ \frac{1}{N - (\tilde{N} + \gamma/\beta)} \right] \left( \frac{1}{2} b^k c \right) \left( \frac{2\kappa - 1}{\kappa - 1} \right) (z^*)^{-\kappa} \end{aligned} \quad (33)$$

$$E(\pi) = \frac{1}{4} L Q \gamma b^k c \left( -\frac{\kappa}{\kappa - 1} - 2 - \frac{\kappa}{\kappa + 1} \right) (z^*)^{-\kappa} = w f_e \quad (34)$$

$$R = \frac{1}{4} L Q \gamma b^k c \left( \frac{\kappa}{\kappa + 1} - \frac{\kappa}{\kappa - 1} \right) (z^*)^{-\kappa} = w L \quad (35)$$

$$Q = y(z^*)^\kappa \left[ \frac{1}{4} \gamma b^k c \left( \frac{\kappa}{\kappa + 1} - \frac{\kappa}{\kappa - 1} \right) \right]^{-1} \quad (36)$$

Combining the last two equations (market clearing and the aggregate demand shifter  $Q$ ) implies that  $y = w$ .  $N$  is the number of consumed goods (as usual equal to:  $(\frac{b}{z^*})^\kappa$ )<sup>2</sup> and  $\tilde{N}$  is “all possible varieties.” To get at welfare, the expenditure function can be derived using the definition of the choke price:

$$e_2(\mathbf{p}) = \left[ \frac{1}{4} \left( \frac{\kappa}{\kappa + 1} - \frac{\kappa}{\kappa - 1} \right) c^2 \gamma b^k (z^*)^{-k-1} \right]^{1/2} \quad (37)$$

**Minimum Quality Requirement** The profit function can be used to compute expected profits, integrating across firms with quality  $z \in (\bar{z}, \infty)$ . Expected profits and aggregate revenues as a function of  $g$  (where  $g = \frac{z^*}{\bar{z}}$ ), are:

$$E[\pi] = \frac{1}{4} Q c \gamma \kappa b^k (z^*)^{-k} \left[ -\frac{1}{\kappa - 1} g^{\kappa-1} - \frac{2}{\kappa} g^\kappa - \frac{1}{\kappa + 1} g^{\kappa+1} \right] \quad (38)$$

$$R = \frac{1}{4} Q c \gamma \kappa b^k (z^*)^{-k} \left[ \frac{1}{-\kappa + 1} g^{\kappa-1} + \frac{1}{\kappa + 1} g^{\kappa+1} \right] \quad (39)$$

Finally, the expenditure function allows us to solve for  $Q$  and utility. The expenditure function can be calculated using the solution for prices and quantity:

$$\begin{aligned} e_2(\mathbf{p}) &= P \left[ \gamma \int \left[ \frac{1}{z} \left( \frac{p(z)}{P} \right)^2 - \left( \frac{p(z)}{P} \right) \right] dz \right]^{1/2} \\ &= c \left[ \left( \frac{1}{4} \gamma \kappa b^k (z^*)^{-\kappa-1} \right) \left( \frac{1}{-\kappa + 1} g^{\kappa-1} + \frac{1}{\kappa + 1} g^{\kappa+1} \right) \right]^{1/2} \end{aligned} \quad (40)$$

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<sup>2</sup>We normalize  $J = 1$  as the number of firms that pay the fixed entry cost since we know this will be a fixed proportion of the market size.

Free entry implies  $E[\pi] = wf_e$ , and we assume that  $w = y$ . The aggregates above are a function of  $Q$ , a demand shifter that only affects the level of demand and is solved such that the budget constraint holds. Using the budget constraint,  $Q = \frac{uP}{e_2(\mathbf{p})}$ , setting  $u = 1$ , and plugging into the expected profit condition, allows us to solve for  $z^*$  in the free entry condition:

$$(z^*)^{k+1} = \frac{\gamma\kappa b^\kappa}{4wf_e} c^2 \frac{1}{e_2(\mathbf{p})} \left[ -\frac{1}{\kappa-1}g^{\kappa-1} - \frac{2}{\kappa}g^\kappa - \frac{1}{\kappa+1}g^{\kappa+1} \right]. \quad (41)$$

Notice that unlike the addilog model, this cutoff is a function of the choke price and therefore changes with  $g$ . Having solved for this cutoff, we plug it into the expenditure function so that:

$$\frac{y}{e_2(\mathbf{p})} = \left( \frac{y}{f_e} \right)^{1/2} \left[ \frac{\left( -\frac{1}{\kappa-1}g^{\kappa-1} - \frac{2}{\kappa}g^\kappa - \frac{1}{\kappa+1}g^{\kappa+1} \right)}{\left( \frac{1}{-\kappa+1}g^{\kappa-1} + \frac{1}{\kappa+1}g^{\kappa+1} \right)} \right]. \quad (42)$$

The above equation implies that the optimal standard ( $g$ ) is equal to 1.

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