Quality Heterogeneity and Misallocation: The Welfare Benefits of Raising your Standards
The CES Case

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In this addendum, we examine the welfare effects of a quality standard under CES preferences, which are not included in Generalized Translated Power (GTP) preferences. In particular, we consider the Benassy-CES (Benassy, 1996) preferences, which allow for the presence of a negative or positive externality associated with the number of varieties available for consumption. In particular, we consider a closed economy of size L, where the representative consumer has the following utility function:

$$U_j = N_j^{\alpha} \left[\int_{\Omega_j} q(\omega)^{\frac{\sigma-1}{\sigma}} z(\omega)^{\frac{1}{\sigma}} d\omega \right]^{\frac{\sigma}{\sigma-1}} \tag{1}$$

where $q(\omega)$ is the quantity consumed of variety ω, $z(\omega)$ is a demand shifter interpreted as quality, N_j is the mass of varieties available for consumption, σ is the elasticity of substitution, and α is a parameter that captures whether diversity is a public good ($\alpha > 0$) or bad ($\alpha < 0$). However, we require $\alpha + \frac{1}{\sigma-1}$ to be positive in order to have love for variety. The aggregate inverse demand for variety ω is given by:

$$p(\omega) = \frac{L}{A} \left(\frac{z(\omega)}{x(\omega)} \right)^{\frac{1}{\sigma}} \tag{2}$$

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where A is a quantity aggregator that equals:

$$A = \int_{\Omega_j} x(\omega)^{\frac{\sigma-1}{\sigma}} z(\omega)^{\frac{1}{\sigma}} d\omega = \left[\frac{LU}{N_j^{\alpha}}\right]^{\frac{\sigma-1}{\sigma}} \tag{3}$$

and $x(\omega) = Lq(\omega)$ is aggregate demand.

The firms' problem is identical to the baseline case, with the exception of the presence of a fixed cost of operation f – independent from the level of the standard. The fixed cost is required because the marginal utility is not bounded from above, in contrast to the GTP case. Profit maximization yields the following optimal quantity and prices:

$$x(z) = \left[\frac{L(\sigma-1)}{A\sigma}\right]^{\sigma} c^{-\sigma} z$$
$$p(z) = \frac{\sigma}{\sigma-1} c$$

Firm revenues $r(z)$ and profits $\pi(z)$ are:

$$r(z) = \frac{\sigma}{\sigma-1}\left[\frac{L(\sigma-1)}{A\sigma}\right]^{\sigma} c^{1-\sigma} z$$
$$\pi(z) = \frac{1}{\sigma-1}\left[\frac{L(\sigma-1)}{A\sigma}\right]^{\sigma} c^{1-\sigma} z - f$$

The market determined quality cutoff z^* is obtained by setting the profits of the cutoff firm to zero $\pi(z^*)$:

$$\frac{1}{\sigma-1}\left[\frac{L(\sigma-1)}{A\sigma}\right]^{\sigma} c^{1-\sigma} z^* = f \tag{4}$$

Using (4), revenues $r(z)$ and profits $\pi(z)$ of firm z become:

$$r(z) = \sigma f \frac{z}{z^*}$$
$$\pi(z) = f\left[\frac{z}{z^*} - 1\right]$$

Moreover, using (3) we can obtain an expression for the indirect utility function:

$$U = z^{*\frac{1}{\sigma-1}} N^{\alpha} \frac{L^{\frac{1}{\sigma-1}}}{c}\left[\left(\frac{\sigma-1}{\sigma}\right)^{\sigma} \frac{1}{f(\sigma-1)}\right]^{\frac{1}{\sigma-1}} \tag{5}$$

The government imposes a standard $\bar{z} \geq z^*$. Moreover, let $g = \frac{\bar{z}}{z^*}$ denote the restrictiveness of the standard. The mass of varieties available for consumption is:

$$N = J\left(\frac{b}{\bar{z}}\right)^{\kappa}$$

where J is the mass of active firms. Market clearing implies that aggregate revenues equal total expenditures, hence:

$$\frac{Jfb^{\kappa}}{\bar{z}^{\kappa}}\left(\frac{\kappa\sigma}{\kappa-1}\right)g = L \tag{6}$$

The zero profit condition implies that expected profits equal the fixed cost of entry f_E:

$$\frac{fb^{\kappa}}{\bar{z}^{\kappa}}\left[\frac{\kappa}{\kappa-1}g-1\right] = f_E \tag{7}$$

Using the zero profit condition (7) to find $\bar{z}$, we can write the market-determined quality cutoff z^* as:

$$z^* = \bar{z}g^{-1} = \left(\frac{fb^{\kappa}}{f_E}\right)^{\frac{1}{\kappa}}\left[\frac{\kappa}{\kappa-1}g-1\right]^{\frac{1}{\kappa}}g^{-1} \tag{8}$$

From the market clearing condition (6), the mass of varieties available for consumption N_j equals:

$$N = \frac{Jb^{\kappa}}{\bar{z}^{\kappa}} = \frac{L(\kappa-1)}{f\kappa\sigma}g^{-1} \tag{9}$$

Using (8) and (9) into the utility function (5) yields:

$$U = \left(\frac{fb^{\kappa}}{f_E}\right)^{\frac{1}{\kappa(\sigma-1)}}\frac{L^{\frac{1}{\sigma-1}+\alpha}}{c}\left[\left(\frac{\sigma-1}{\sigma}\right)^{\sigma}\frac{1}{f(\sigma-1)}\right]^{\frac{1}{\sigma-1}}\left(\frac{(\kappa-1)}{f\kappa\sigma}\right)^{\alpha}\left[\frac{\kappa}{\kappa-1}g-1\right]^{\frac{1}{\kappa(\sigma-1)}-\alpha}g^{-\frac{1}{\sigma-1}-\alpha} =$$

$$= \bar{U}\left[\frac{\kappa}{\kappa-1}g-1\right]^{\frac{1}{\kappa(\sigma-1)}}g^{-\frac{1}{\sigma-1}-\alpha} \tag{10}$$

Taking the derivative of (10) yields:

$$\frac{dU_j}{dg} = \frac{U}{(\sigma-1)g}\left(-1-\alpha(\sigma-1)+\frac{g}{\kappa g+(\kappa-1)}\right) \tag{11}$$

The optimal level of quality requirements is:

$$g^{opt} = 1 - \frac{\alpha(\sigma-1)}{\kappa(\sigma-1)\left(\frac{1}{\sigma-1}+\alpha\right)-1} \tag{12}$$

If $\alpha = 0$, the model collapses to a standard Melitz-type model with CES preferences. The resulting allocation in such model is efficient, hence the optimal policy is $g^{opt} = 1$: setting the minimum quality requirement equal to the market determined minimum quality.

If $\alpha > 0$, diversity is a public good. The market determined mass of varieties is not optimal: consumers would prefer a larger diversity and firms do not internalize such externality. In this case, a minimum quality requirement is not able to increase the mass of varieties.

A minimum quality requirement is optimal when $\alpha < 0$, and diversity is a public bad. Consumers still exhibit love for variety, since their love for variety is parametrized by $\alpha + \frac{1}{\sigma-1} > 0$. However, consumers would prefer a lower diversity relative to the market determined one. Introducing a minimum quality requirement improves the market allocation. In particular:

$$\bar{g}^{opt} = \left[1 - \frac{\alpha(\sigma-1)}{\kappa(\sigma-1)\left(\frac{1}{\sigma-1}+\alpha\right)-1}\right] z^* =$$
$$= \left(\frac{fb^{\kappa}}{f_E}\right)^{\frac{1}{\kappa}} \left[\frac{1}{\kappa(\sigma-1)\left(\frac{1}{\sigma-1}+\alpha\right)-1}\right]^{\frac{1}{\kappa}}$$

References

J.-P. Benassy. Taste for variety and optimum production patterns in monopolistic competition. *Economics Letters*, 52(1):41–47, 1996.